Problem of the week

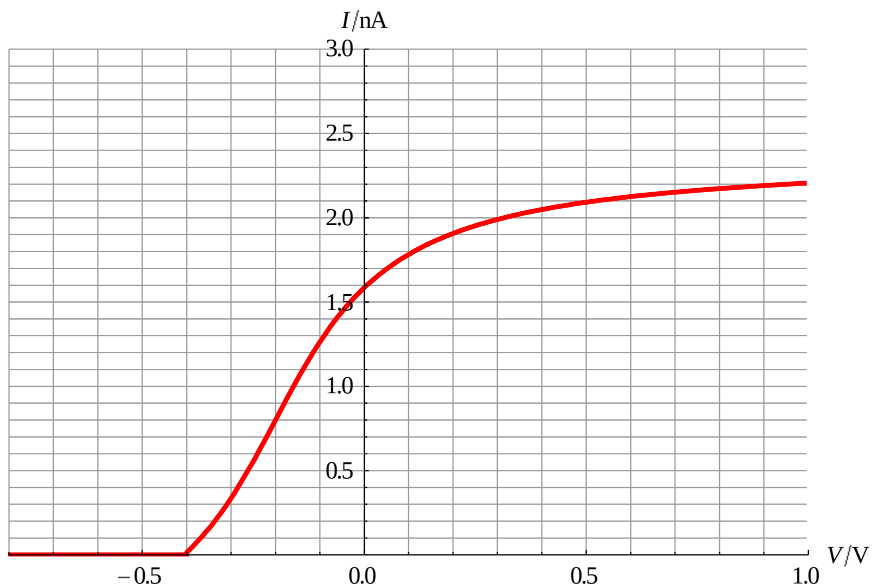
Quantum Physics (HL only)

(a) Very weak intensity light incident on a metallic surface ejects electrons from the surface with no apparent time delay. Explain how this observation

(i) is not consistent with the wave theory of light,

(ii) is consistent with the photon theory of light.

(b) The graph shows the variation with applied voltage of the photocurrent in a photoelectric experiment. The light incident on the surface has wavelength 430 nm.



Determine

(i) the kinetic energy of the emitted electrons,

(ii) the work function of the surface.

(c)

(i) The graph in (b) shows that as the voltage increases the current approaches a constant value. Explain why.

(ii) The intensity of the light incident on the surface is increased but the wavelength stays the same. Suggest any changes to the graph in (b).

(iii) The wavelength of light is decreased but the intensity incident on the surface stays the same. Assuming that each photon causes the emission of one electron state and explain any changes to the graph in (b).

(d)

(i) Calculate the de Broglie wavelength of a proton that has been accelerated from rest by a potential difference of 120 kV.

(ii) Outline the experimental evidence that electrons have wavelike properties.

(iii) Suggest why it is meaningless to assign wavelike properties to a football of mass 0.4 kg moving at 10 m s^{-1}.

(e) A photon of total energy equal to twice the rest energy of an electron is incident on a stationary electron. The photon scatters at an angle of 90° relative to its original direction.

(i) Calculate the wavelength of the incident and of the scattered photon.

(ii) Determine the energy transferred to the electron.

(iii) Calculate the angle at which the electron recoils.

(iv) The stationary electron is replaced by a stationary carbon atom. Suggest why the shift in wavelength of the incident photon will be negligible.

Answers

(a)

(i) In the wave theory the energy carried by the light waves is proportional to the intensity so if the intensity is low very little energy is being transferred. Hence there should be a delay during which energy is being accumulated by the electron.

(ii) In the photon theory the energy is transferred to the electron in one step as the photon is absorbed by the electron. Hence there should be no time delay.

(b)

(i) The stopping voltage is 0.40 V and so the kinetic energy of the electrons is 0.40 eV.

(ii) $E_K = hf - \phi \Rightarrow \phi = hf - E_K = \frac{hc}{\lambda} - 0.40 = \frac{1.24\times 10^{-6}}{430\times 10^{-9}} - 0.40 = 2.48 \approx 2.5 \text{ eV}$.

(c)

(i) As the voltage increases more and more electrons are attracted to the collecting plate. Eventually, **all** the emitted electrons will be collected and hence the current cannot increase any further.

(ii) More electrons will be emitted since more photons are now incident. The stopping voltage will remain the same, so we have a curve beginning at – 0.40 V and rising above the previous curve.

(iii) The intensity stays the same but now each photon carries more energy. For this to happen the number of incident photons must decrease hence the current will be less than before. The curve will be below the original curve and will start to the left of – 0.40 V since the energy of the electrons will be greater.

(d)

(i) The kinetic energy will be 120 keV and so $\frac{p^2}{2m_p} = 120\times 10^3 \times 1.6\times 10^{-19}$

$p = \sqrt{2\times 1.67\times 10^{-27}\times 120\times 10^3\times 1.6\times 10^{-19}} = 8.0\times 10^{-21} \text{ N s}$. Hence

$\lambda = \frac{h}{p} = \frac{6.63\times 10^{-34}}{8.0\times 10^{-21}} = 8.28\times 10^{-14} \text{ m}$.

(ii) The intensity distribution of a beam of electrons scattering off crystal atoms shows a typical diffraction pattern characteristic of wavelike behavior. The wavelength can be estimated from the diffraction pattern and is in agreement with the de Broglie formula.

(iii) The wavelength would be of order $\lambda = \frac{h}{p} = \frac{6.63\times 10^{-34}}{0.4\times 10} \approx 10^{-34} \text{ m}$. For this wavelength to be observable diffraction through apertures of this order of magnitude are required and these do not exist.

(e)

(i) $\lambda_i = \frac{hc}{E} = \frac{hc}{2m_e c^2} = \frac{h}{2m_e c}; \lambda_f - \lambda_i = \frac{h}{m_e c}(1 - \cos 90^\circ) = \frac{h}{m_e c}$. Hence $\lambda_f = \frac{h}{m_e c} + \frac{h}{2m_e c} = \frac{3h}{2m_e c}$.

(ii) The energy of the scattered photon is $E = \frac{hc}{\lambda_f} = \frac{hc}{\frac{3h}{2m_e c}} = \frac{2}{3}m_e c^2$. The loss in energy of the photon is therefore $\Delta E = 2m_e c^2 - \frac{2}{3}m_e c^2 = \frac{4}{3}m_e c^2$ and this is the kinetic energy gained by the electron.

(iii) Let *p* be the magnitude of the electron's momentum after recoiling. The incident photon's momentum is $\frac{E}{c} = 2m_e c$. The scattered photon momentum is $\frac{2m_e c}{3}$.

Conservation of momentum gives:

$$2m_e c = 0 + p\cos\phi$$

$$0 = \frac{2m_e c}{3} - p\sin\phi$$

Then

$p^2 = (p\cos\phi)^2 + (p\sin\phi)^2 = (2m_e c)^2 + \left(\frac{2m_e c}{3}\right)^2 = \frac{40}{9}(m_e c)^2$ so that $p = \frac{\sqrt{40}}{3}m_e c$.

Further, $\tan\phi = \frac{\frac{2m_e c}{3}}{2m_e c} = \frac{4}{3}$ giving $\phi = 53^\circ$.

(iv) The shift in wavelength would now be $\lambda_f - \lambda_i = \frac{h}{Mc}(1 - \cos 90^\circ) = \frac{h}{Mc}$ where *M* is the mass of the carbon atom. Compared to the incident photon wavelength of $\frac{h}{2m_e c}$ we have $\frac{\frac{h}{Mc}}{\frac{h}{2m_e c}} = \frac{2m_e}{M} = \frac{2 \times 5 \times 10^{-4}\,u}{12\,u} \approx 10^{-4}$ and so negligible.